THE INFLUENCE OF JOINT AERATION ON DYNAMIC UPLIFT OF CONCRETE SLABS OF PLUNGE POOL LININGS

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Abstract : This paper presents the influence of free air in joints of concrete slabs of plunge pool linings on the pressure wave propagation in the joints and on the resulting net dynamic impulsions on the slabs. A general philosophy of pressure wave propagation in joints is procured, together with a simple analytical model showing the influence of the wave celerity on the net dynamic impulsions on the slabs. Furthermore, a two-phase transient model is used to compute the pressures and impulsions on a slab for a sinusoidal pressure signal as input. A comparison with pressures measured on near-prototype scaled experimental tests is provided.

Keywords: joint aeration, transient pressures, uplift of concrete slabs

1. INTRODUCTION

The presence of free air in artificially simulated joints under the impact of high-velocity jets was found to have a considerable influence on the transient pressure pattern inside the joints (Bollaert, 2002a & 2002b; Bollaert & Schleiss 2001). The air makes the flow mixture in the joint highly compressible and causes a significant decrease in celerity of the pressure waves that are generated by the impacting jet. The cyclic pressure excitation of the jet at the entrance of the joint can, under certain circumstances, be capable to create pressure amplification and resonance effects in the joint. This depends on the celerity, the total length of the joint and the end conditions of the joint, conform the principles of a resonator system.

Furthermore, the exact amount of free air in the joint, and thus also the exact wave celerity, changes continuously as a function of the instantaneous pressure values in the joint. This makes the pressure wave propagation in the joint highly non-linear and causes the appearance of peak pressures, which completely modify the pressure field and corresponding impulsions.

In the following, this principle has been applied to joints underneath concrete slabs of plunge pool linings of high-head dams, in order to point out the influence on possible failure of the slabs by dynamic uplift. Dynamic uplift is caused by the built-up of a net difference in pressures over and underneath the slab. The persistency (integration) with time of this difference in pressures defines the value of the net uplift impulsion on the slab. This net impulsion is able to eject the slab from its surroundings (Fiorotto & Rinaldo 1992). Well-known cases are failures at Malpaso Dam (Mexico), Tarbela Dam (Pakistan) and Karnafuli Dam (Pakistan). Other cases of violent pressure wave propagation in joints have been noticed in the field of rock cliff erosion by dynamic wave impact (Müller 1997).

First, it will be shown that the presence of air generates net dynamic uplift impulsions on the slabs that are completely different than for pure water. Then, the non-linear character of the pressure wave propagation, which completely modifies the pressures in the joint and the net uplift impulsion on the concrete slabs, is highlighted by use of a numerical model.

2. PRESSURE WAVE PROPAGATION IN JOINTS

PHILOSOPHY

Pressure wave propagation in joints is largely influenced by the boundaries, which can reflect the incoming waves. The adopted approach is based on the simplest cases, i.e. open or closed-end boundaries. These are similar to open or closed-end rock joints and joints underneath concrete slabs. Jet impact on a joint has all elements of a resonator system: the jet provides a periodic excitation and the joint is a resonance chamber. The boundaries are formed by the rock and/or the concrete slabs. The periodic nature of the jet excitation is defined by its spectral content. This causes resonance effects whenever part of the spectral content (turbulent eddies) of the jet is situated near the natural frequencies of the joint. At each cycle, additional energy can so be injected into the system. When this periodical energy injection is higher than the periodical energy dissipation, resonance conditions might occur.

These conditions typically happen at or near the theoretical natural frequencies or eigen-frequencies of the system in question. For joints in rock or in between concrete slabs of plunge pool linings, two main boundary systems can be distinguished:

- the *open-closed boundary system*, relevant to joints that are not fully formed, $\lambda/4$ – resonator, with resonance frequencies at $f_{res} = (1+2n)\cdot(c/4L)$, n = 0, 1, 2, …
- the *open-open boundary system*, for joints formed by distinct rock blocks or concrete slabs, $\lambda/2$ – resonator, with resonance frequencies at $f_{res} = (n)\cdot(c/4L)$, n = 1, 2, 3…

in which c is the wave celerity and λ is the wavelength (=c/f). This is presented in Figure 1. Due to the compressibility of the flow mixture inside the joint, an infinite number of modes of oscillation exist, just like the vibrations of a mechanical system with an infinite number of masses and springs. The first mode of vibration is the fundamental or first harmonic; the others are higher harmonics. An open boundary corresponds to a pressure node; a closed boundary is an antinode. For the open-closed system, there is a pressure node during even harmonics (2^{nd}, 4^{th}, …) and an antinode during odd harmonics (1^{st}, 3^{rd}, …). For the open-open system, two pressure nodes always exist. The exact location of nodes and antinodes depends upon the harmonic at which the system is oscillating.

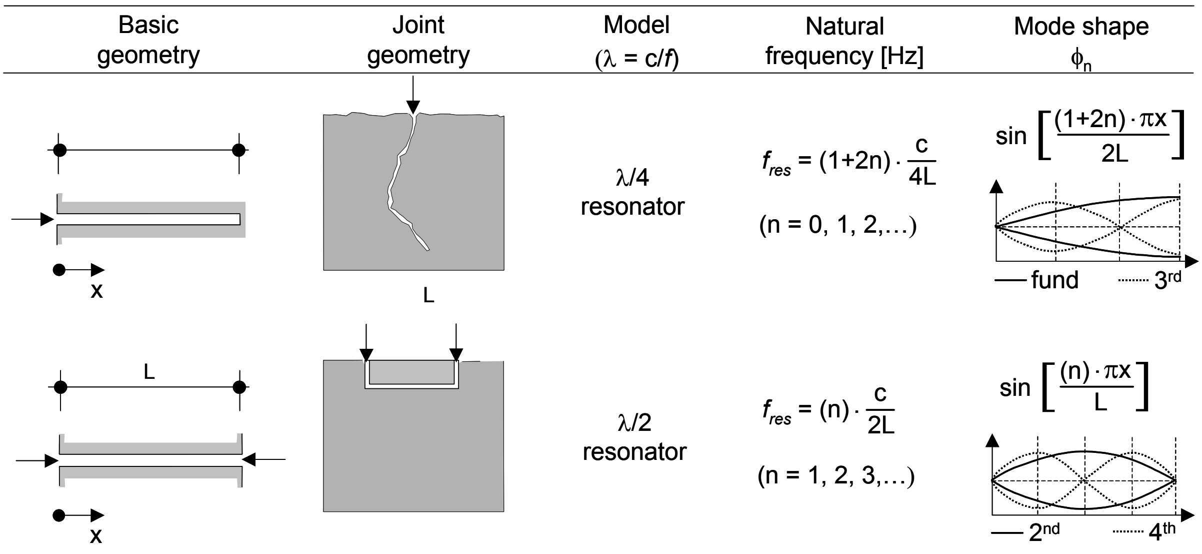


Figure 1 Two basic joint configurations with well-defined boundaries. The terminations are considered to be perfectly weak (open boundary) or perfectly rigid (closed boundary).

SIMPLIFIED MODEL SHOWING CELERITY INFLUENCE

Forces on a concrete slab

A simplified and comprehensive analytical model is used to point out the influence of the pressure wave celerity on the net dynamic impulsion on concrete slabs. The net impulse $I_{\Delta tpulse}$, corresponding to a pressure pulse of duration Δt_{pulse}, is defined by integrating the force equilibrium at every time step of the pulse. This time step is small so that no significant pressure gradients occur. This results in an uplift velocity $V_{\Delta tpulse}$. The net impulse is written:

$$I_{\Delta tpulse} = \int_0^{\Delta tpulse} \left(F_u - F_o - G_b - F_{sh}\right) \cdot dt = m \cdot V_{\Delta tpulse} \quad (1)$$

in which m stands for the mass of the slab, F_u and F_o the pressure forces under and over the slab, G_b the immerged weight of the slab and F_{sh} eventual shear forces along the joint surfaces. The pressure distributions above and underneath the slab are spatially integrated. However, due to violent transient effects, the pressure gradient with time is generally much higher than the pressure gradient with space. Therefore, as a first approximation, a space-averaged value is chosen. The kinetic energy given to the slab is simply transformed into potential energy as a function of the mass of the slab. The total uplift displacement h_{up} and the mass m of the slab are written:

$$V_{\Delta tpulse} = \sqrt{2 \cdot g \cdot h_{up}} \quad (2) \qquad\qquad m = \rho \cdot \text{Volume} = \rho \cdot (x \cdot y \cdot z) \quad (3)$$

in which ρ stands for the density of the concrete and x, y and z respectively for the longitudinal, transversal and vertical dimensions of the slab. Furthermore, the weight of the slab and the pressure forces over and under the slab directly depend on the horizontal surface, i.e. $x \cdot y$. Therefore, when neglecting the shear forces F_{sh}, which depend on the vertical length z, this product can be eliminated from both the left and right hand side of eq. (1). As a result, the uplift velocity $V_{\Delta tpulse}$ is inversely proportional to the height of the slab z. According to eq. (2), the uplift displacement h_{up} of a slab is inversely proportional to the square of its height z^2. This shows that failure criteria based on uplift of concrete slabs are considerably influenced by the height of the slab. A simple computation demonstrates this dependence.

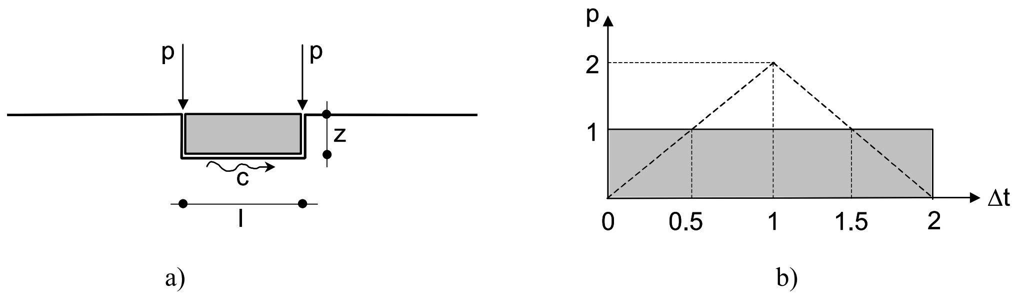


Figure 2 Pressure pulses applied underneath a concrete slab of height z and length l: a) geometrical situation, b) triangular shape and rectangular simplification of the pressure pulse.

Dynamic pressure field underneath the slab

Assuming a two-dimensional problem, the form factor of the slab can be defined as the ratio of the height to the horizontal extension z/l. The abovementioned philosophy has been followed for a constant pressure head of 50 and 100 m. This pressure is applied underneath a slab of height z and length l, at both joint entrances, as indicated in Figure 2a. The pressures over the slab are neglected for sake of simplicity. Furthermore, the time period of application of the underpressure is defined in accordance with Figure 2b. Each of the pulses has a value p and a time duration $\Delta t = (l/c)$, thus neglecting the height z in the total joint length. Due to superposition of the pulses under the slab, the total time duration is equal to 2Δt and of triangular shape. In the following, this triangular shape has been simplified to a rectangular one with the same surface (= impulsion) but for a constant pressure value p.

Figure 3a presents the influence of the form factor z/l on the non-dimensional uplift displacement h_{up}/z for a slab length l of 3 m. The impulse on the slab has been calculated for two wave celerities: 600 m/s and 1'000 m/s. The celerity has a significant influence on the time duration of the pressure pulse and thus on the uplift velocity that is finally given to the slab. Figure 3b shows the same relationship, but for a length that is equal to one third of the initial one, i.e. 1 m. The concrete slab is ejected more easily, but neglecting the side lengths is of importance in this case. Again the significance of the celerity is pointed out. The minimum uplift displacement h_{cr} necessary for ejection is assumed equal to the height of the slab z.

It can be concluded that the form factor z/l is of primary importance on the ejection of the slab. Secondly, within the limits of this simple model, the absolute dimensions of the slab seem also to be of significance. This, of course, under the assumption that the surface pressure is neglected. The importance of the form factor decreases for decreasing slab dimensions.

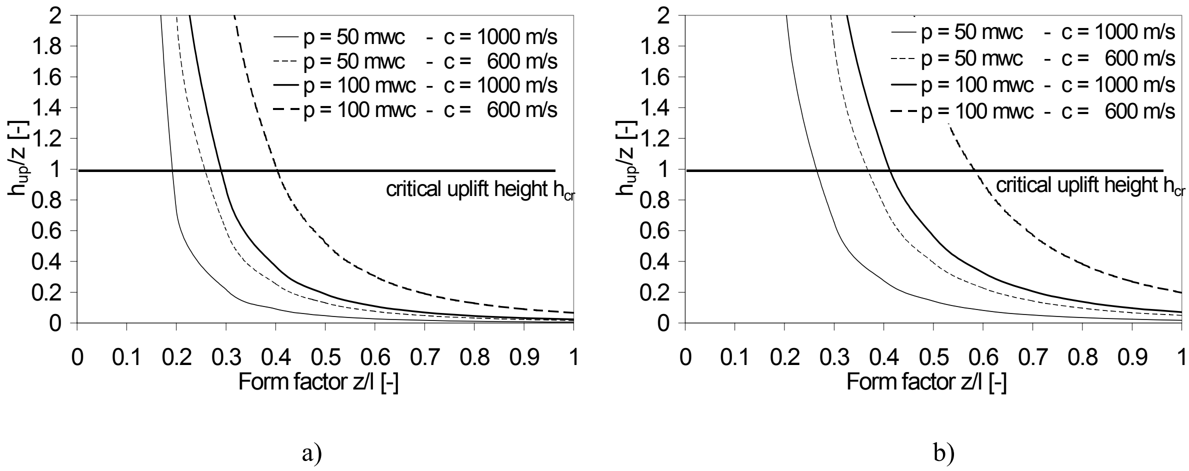


Figure 3 Non-dimensional concrete slab displacement as a function of the form factor z/l and for different pressure wave celerities c: a) concrete slab of length l = 3 m, b) concrete slab of length l = 1 m.

TWO-PHASE TRANSIENT MODEL FOR DYNAMIC UPLIFT OF SLABS

Basic equations

A basic numerical model of pressurized flow in 1D joints uses the transient flow equations for a homogeneous two-phase fluid. This means that the air-water mixture is simulated as a pseudofluid with average properties and, thus, only one set of conservation equations. The compressibility and the pressure wave celerity c of the mixture strongly depend on the volume of free air in the liquid. Therefore, when neglecting any exchange between the phases in the conservation equations, a constitutive relationship between the wave celerity c and the pressure p has to be added. The mass and momentum equations of the pseudofluid are expressed as follows (Bollaert, 2002b):

$$\frac{\partial p}{\partial t} + \frac{c^2}{g} \cdot \frac{\partial V}{\partial x} = 0 \qquad (4)$$

$$\eta \cdot \frac{\partial V}{\partial t} + \frac{\partial}{\partial x}\left(\beta V^2\right) + g \cdot \frac{\partial p}{\partial x} + \lambda \cdot V = 0 \qquad (5)$$

in which p is the pressure head (m), V the mean velocity (m/s), and c the wave celerity (m/s). The terms λ, η and β account for steady, unsteady and uneven velocity distribution friction losses. They incorporate all possible energy losses and cannot be compared with the Darcy-Weisbach friction term that is usually applied for one-phase steady-state flow. Especially the value of λ is often quite different, due to the particular damping effect generated by the two-phase transient character of the flow. Further details can be found in Bollaert (2002b).

The form of the constitutive equation that relates the celerity c with the pressure p is defined by physical laws that describe the volume and the quantity of free air in a liquid as a function of pressure (ideal gas law, Henry's law). This background will not be outlined herein. Based on c-p relationships as derived from prototype-scaled experimental tests (Bollaert 2002b; Bollaert & Schleiss 2001), it is assumed that a quadratic relationship is appropriate:

$$c(t) = k_1 + k_2 \cdot p(t) + k_3 \cdot p^2(t) \tag{6}$$

in which k_1, k_2 and k_3 are parameters defined based on experimental data.

Celerity-pressure relationships

Appropriate celerity-pressure relationships have been derived from prototype-scaled experimental tests and from numerical optimizations of the k-parameters in eq. (6). This has resulted in relations 1 and 2 at Table 1 and in Figure 4:

parameters	**k_1**	**k_2**	**k_3**
c-p relation 1	20	100	3
c-p relation 2	0	50	0
c-p relation 3	200	0	0

Table 1: Parameters of the c-p relations as derived from measured data

Relation 1 corresponds to measured data in closed -end joints, for jet impact velocities close to prototype values (up to 30 m/s) and with considerable air entrainment in the plunge pool and in the joints. Relation 2 also represents high jet velocities and air entrainment and was measured for open-end joints (underneath slab). In the following, these relationships are presented and a comparison is made with the assumption of a constant wave celerity of 200 m/s, i.e. c-p relation 3.

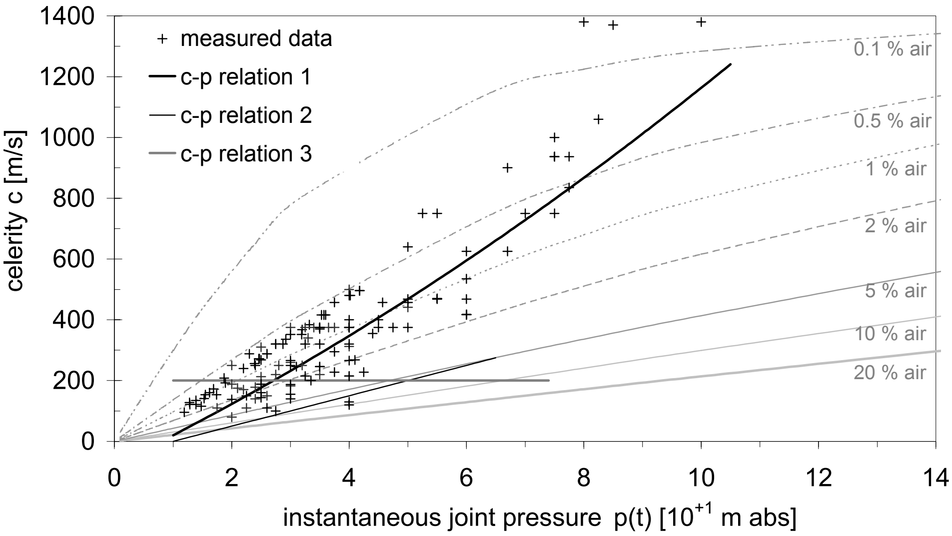
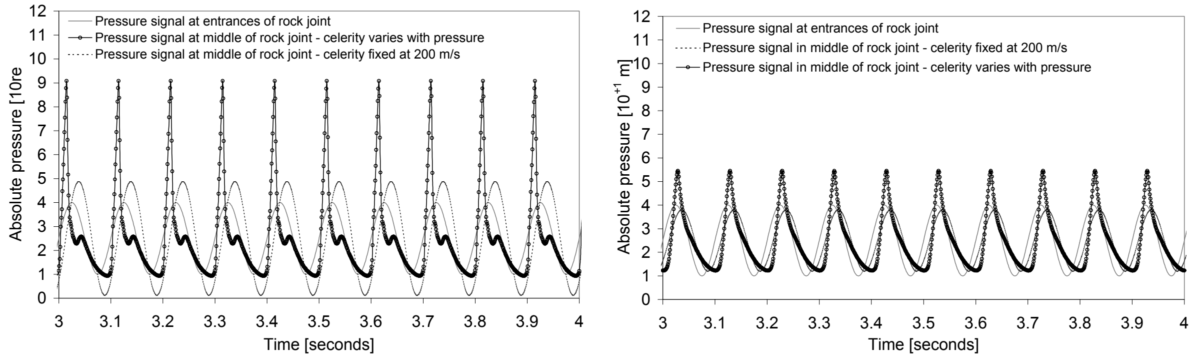


Figure 4: Measured data and derived c-p relationships

Comparison of variable and constant celerity-pressure relationships
A comparison is made between the different c-p relations as presented at Table 1 and in Figure 4, by applying a simple sinusoidal pressure signal at both joint entrances of a concrete slab following Figure 2a. This signal has a frequency of 10 Hz and represents plausible pressure values fluctuating between 10 and 40 m of absolute pressure head (Bollaert 2002b). The total length of the joint L is assumed 8 m. For a constant wave celerity c = 200 m/s, this results in a theoretical fundamental resonance frequency f_{res} = 200/(2x8) = 12.5 Hz (Figure 1), i.e. close to the frequency of the incoming pressure signal.

First of all, a comparison is made between relation 1 and relation 3 (Table 1) assuming that there is no phase difference between the pressures that enter the joint, i.e. the two pulses are perfectly in-phase (phase difference $\phi = 0$). Furthermore, the λ-parameter in eq. (5) is equal to 0.25 (Figure 5a) and 0.50 (Figure 5b). These were found to be plausible friction values for impact velocities of 10-25 m/s (Bollaert 2002b). The influence of β and η is neglected.



a) b)

Figure 5: Comparison of c-p relation 1 with c-p relation 3 for a sinusoidal pressure signal at the joint entrances: a) friction factor $\lambda = 0.25$; b) friction factor $\lambda = 0.50$.

The c-p relation 1, expressing a celerity that varies with pressure, results in highly peaked pressures during a short time interval, while the constant celerity assumption (c-p relation 3) leads to a pressure signal that is very close to the input pressure signals. Especially for higher friction values, the non-linear assumption of variable celerities will result in higher net impulsions on the concrete slabs than the constant celerity assumption. The pressures obtained at constant celerity and for a friction factor of 0.25 are unrealistic because they are less than the atmospheric pressure (10 m of pressure head), which is physically not plausible given the high air content and thus the absence of cavitation effects. Therefore, their corresponding net impulsion is unrealistic.

Secondly, Figures 6a & 6b present the same situation but for a phase difference of $\phi = \pi/2$ between the incoming pressure pulses at the joint entrances. The assumption of a constant celerity (c-p relation 3) generates no noticeable impulsion on the slab, whereas the non-linear assumption (c-p relation 1) results in a peaked net uplift pressure and impulsion. This is particularly valid at low to moderate friction factors and decreases with increasing friction.

Finally, for a phase difference of $\phi = \pi$ (Figures 6c & 6d), only the assumption of variable celerity (c-p relation 1) generates some net uplift pressure and impulsion on the slab. This impulsion, however, seems to be rather insignificant compared to the aforementioned cases.

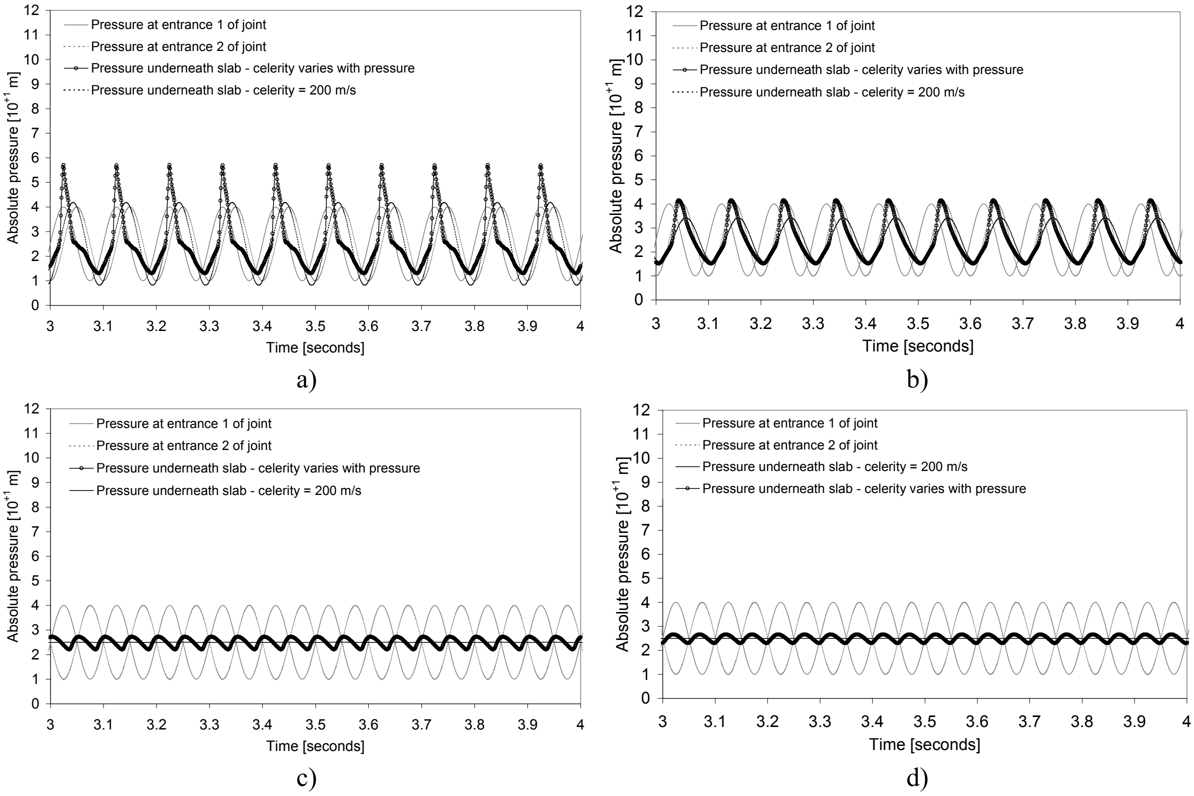


Figure 6: Comparison of c-p relations for a sinusoidal pressure at the joint entrances and a phase difference of: a) $\phi = \pi/2$ and $\lambda = 0.25$; b) $\phi = \pi/2$ and $\lambda = 0.50$; c) $\phi = \pi$ and $\lambda = 0.25$; d) $\phi = \pi$ and $\lambda = 0.50$.

Comparison of measured and computed pressures underneath slab

Finally, the air-water numerical model has been verified by applying pressures at the joint entrances that have been measured during experimental tests (Bollaert 2002b). During these measurements, the pressure underneath the simulated slab has been recorded simultaneously with the surface pressures and a direct comparison between measured and computed values can be made. The measured c-p relation for this test case is relation 2, involving a lot of free air in the joint (10 to 20 %). The jet impact velocity is 25 m/s and the friction factor $\lambda = 0.25$.

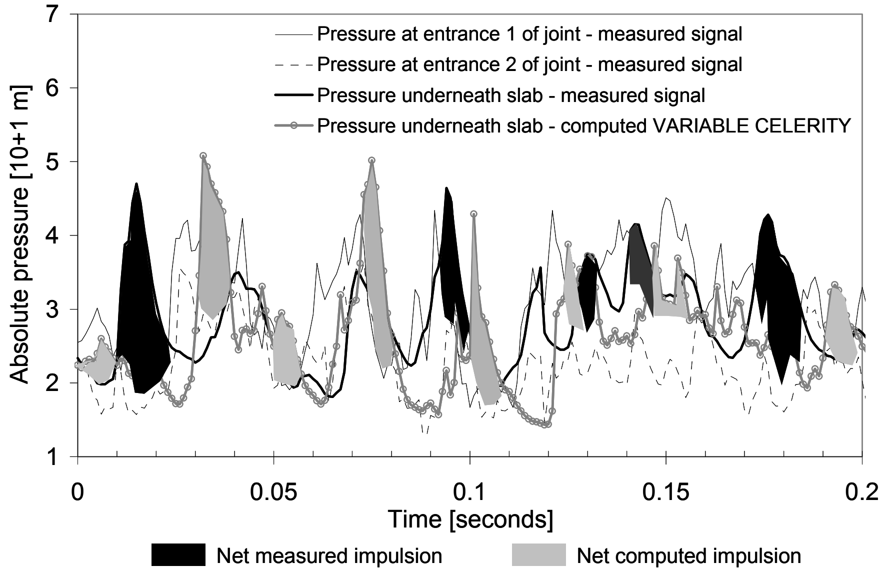


Figure 7: Comparison of measured and computed pressures underneath concrete slab.

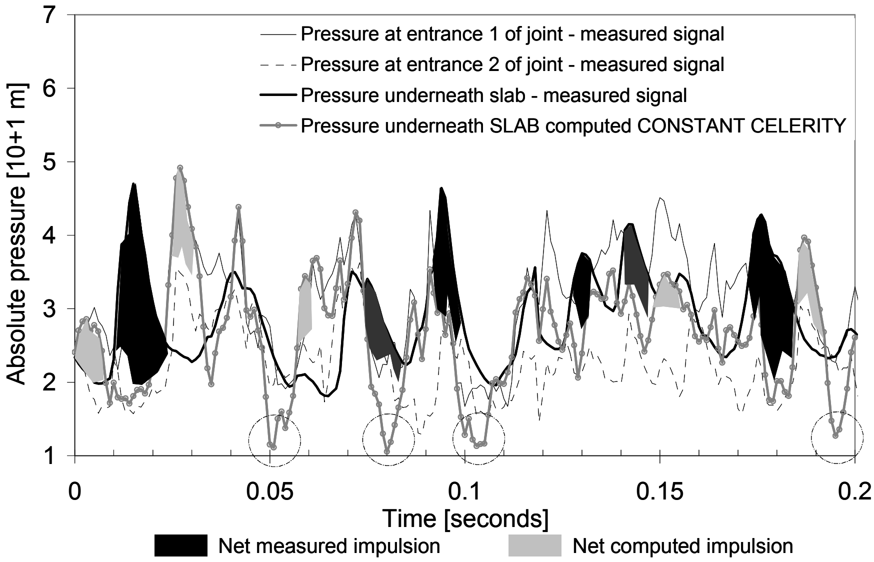


Figure 8: Comparison of measured and computed pressures underneath concrete slab.

Figure 7 shows that, for a variable celerity (relation 2), the computed pressures and corresponding net impulsions (represented by grey and black surfaces) are in quite good agreement with the measured data. The general shape and values of the peak pressures are correctly generated. However, not all pressure peaks are apparent or at the right moment, due to the fact that the measured pressure signals at the joint entrances could not be made exactly at the entrances but somewhat aside (for technical reasons). This automatically induces errors and time lags between measurements and computations. Figure 8 presents the same information but for a constant celerity (relation 3). The measured pressures and impulsions are poorly reproduced by the computations, and pressure spikes appear that are physically not plausible.

3. CONCLUSIONS

Prototype-scaled measurements of pressures in joints underneath concrete slabs were found to be significantly influenced by the presence of free air. A simple analytical model determining the net impulsions on a slab indicates that the celerity of the pressure waves is of great significance, as well as the variation of the celerity with pressure. Based on a two-phase transient numerical computation, it has been shown that the use of a constant celerity as a function of pressure poorly reproduces the measured pressure peaks, spikes and impulsions on the slabs. Introducing a variation of the celerity with pressure, based on physical laws, procures a much better agreement with the measured data. It is believed that the present information is of direct significance to design criteria for the safety of concrete slabs of plunge pool linings.

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