ERODIBILITY OF FRACTURED MEDIA: CASE STUDIES

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A new engineering model, the Comprehensive Scour Model (CSM), has been developed for evaluation of erosion in fractured media, such as rock, concrete or clays (Bollaert, 2004). The model is physically-based and relies on break-up by progressive fracturing of existing discontinuities as well as on subsequent dynamic ejection of so formed loose elements. It is not only able to predict the ultimate state of scour, but also the time evolution of the phenomenon. In the following, the model is applied to two completely different flow situations: the scour formation of the plunge pool downstream of Kariba Dam (Zimbabwe) and the potential erosion around bridge piers founded in shaled-rock on the Mississippi River and tested at the University of Iowa (USA, Nakato 2002).

1 Introduction
A comprehensive model to evaluate scour of fractured media has been developed. The model is based on a parametric description of the main physical processes that are responsible for scour. The model parameters are chosen in a way to enhance and simplify applications, without compromising basic physical laws. The main processes responsible for scour applied to a plunge pool behind a dam are presented in Figure 1.

Figure 1. Physical-mechanical processes of scour of fractured media, such as rock in a plunge pool.

A high-velocity plunging jet diffuses through the pool and generates a turbulent shear layer. The impact of this shear-layer at the bottom results in dynamic pressure fluctuations. These may enter underlying joints and progressively break them open, until the joints encounter each other. Then, instantaneous net pressure differences over and under the formed blocks may eject them from the surrounding mass. The blocks may be further
broken-up by re-circulation in the plunge pool (ball-milling), or transferred to the downstream river. The present scour model focuses on fracturing of joints by water pressure fluctuations and on dynamic ejection of single blocks by net uplift pressures.

2 Comprehensive Scour Model (CSM)

The Comprehensive Scour Model (Bollaert, 2004) comprises two methods that describe failure of jointed media. The first, the Comprehensive Fracture Mechanics (CFM) method, determines the ultimate scour by expressing brittle or time-dependent joint propagation due to water pressures. The second, the Dynamic Impulsion (DI) method, describes ejection of blocks from their mass due to sudden uplift pressures. The structure of the model consists of three modules: the falling jet, the plunge pool and the fractured medium. The latter implements the two aforementioned failure criteria.

2.1 Falling jet module

This module describes how the characteristics of the jet are transformed from dam issuance to plunge pool (Fig. 1). Three parameters characterize the jet at issuance: the velocity \( V_i \), the diameter (or width) \( D_i \) and the initial turbulence intensity \( T_u \), defined as the ratio of velocity fluctuations to mean velocity. The jet trajectory is based on ballistics and air drag and is not outlined further. The jet module computes the longitudinal location of impact, the total trajectory length \( L \) and the velocity and diameter at impact \( V_j \) and \( D_j \). The turbulence intensity defines the spread of the jet \( \delta_{out} \) (Ervine et al. 1997). Typical outer angles are 3-4 %. The corresponding inner angles of spread are 0.5 - 1 %. Superposition of outer spread to initial jet diameter \( D_i \) results in the outer jet diameter \( D_{out} \), used to determine the extent of the zone at the bottom where severe pressure damage may occur. Relevant mathematical expressions can be found in Bollaert (2004).

2.2 Plunge pool module

This module describes the hydraulic and geometric characteristics of the jet when traversing the plunge pool and defines the water pressures at the bottom of the fractured medium. The water depth \( Y \) is essential. For near-vertically impacting jets, it is defined as the difference between the water level and the bottom at the point of impact. The water depth increases with discharge and scour formation. Initially, \( Y \) equals the tailwater depth \( t \) (Figure 1). During scour formation, \( Y \) has to be increased with the depth of the formed scour \( h \). The water depth \( Y \) and jet diameter at impact \( D_j \) determine the ratio of water depth to jet diameter at impact \( Y/D_j \). This ratio is directly related to jet diffusion. Dynamic pressures acting at the bottom can be generated by core jets, for small water depths \( Y \), or by developed jets, appearing for \( Y/D_j \) higher than 4 to 6 (for plunging jets). The most relevant pressure characteristics are the mean dynamic pressure coefficient \( C_{pa} \) and the root-mean-square (rms) coefficient of the fluctuating dynamic pressures \( C'_{pa} \), both measured directly under the centerline of the jet. These coefficients correspond to the ratio of pressure head (in [m]) to incoming kinetic energy of the jet \( (V^2/2g) \) and can be found in Bollaert (2004). The rms coefficients depend on the initial turbulence intensity \( T_u \) of the jet at issuance. Typical prototype values for \( T_u \) are around 4-5 %.
2.3 Fractured medium module

Pressures at the bottom are used for determination of pressures inside joints. The parameters are: 1) maximum dynamic pressure $C_{pa}^{\text{max}}$, 2) amplitude of pressure cycles $\Delta \rho$, 3) frequency of pressure cycles $f_c$ and 4) maximum dynamic impulsion $C_{pa}^{\text{max}}$. The 1st parameter is relevant to brittle propagation of joints. The 2nd and 3rd parameters express time-dependent propagation of joints. The 4th parameter defines uplift of blocks. $C_{pa}^{\text{max}}$ is obtained through multiplication of $C_{pa}$ with an amplification factor $\Gamma$, and by superposition with $C_{pa}$. $\Gamma$ expresses the ratio of peak value inside the joint to rms value of pressures at the bottom. The maximum pressure is written as:

$$P_{\text{max}} [\text{Pa}] = \gamma \cdot C_{pa}^{\text{max}} \cdot \frac{V^2}{2g} = \gamma \cdot \left( C_{pa} + \Gamma \cdot C_{pa} \right) \cdot \frac{V^2}{2g}$$

(1)

The frequency of the pressure cycles $f_c$ follows the assumption of a perfect resonator system and depends on the air concentration in the joint $\alpha_i$ and on the length of the joint $L_f$. For practice, a first hand estimation for $f_c$ is 50 to 200 Hz, considering a mean wave celerity of 200 to 400 m/s and joint lengths of 0.5 to 1 m.

Second, the resistance of the fractured medium has to be determined. The cyclic character of the pressures makes it possible to describe joint propagation by fatigue stresses occurring at the tip of the joint. This can be done by Linear Elastic Fracture Mechanics (LEFM). A simplified methodology is proposed (Bollaert, 2004). It is called the Comprehensive Fracture Mechanics (CFM) method and is applicable to any partially jointed medium. Pure tensile pressure loading inside joints is described by the stress intensity factor $K_{I}$, which represents the amplitude of stresses generated by water pressures at the tip of the joint. The corresponding resistance of the medium against joint propagation is expressed by its fracture toughness $K_{Ic}$.

Joint propagation distinguishes between brittle and time-dependent propagation. The former happens for a stress intensity higher than the fracture toughness of the material. The latter is occurring for a stress intensity inferior to the material’s resistance. Joints may then propagate by fatigue, which depends on the frequency and amplitude of the load cycles. The stresses in the medium are characterized by $K_I$:

$$K_I = P_{\text{max}} \cdot F \cdot \sqrt{\pi \cdot L_f}$$

(2)

in which $K_I$ is in MPa$\sqrt{\text{m}}$ and $P_{\text{max}}$ in MPa.
The boundary correction factor $F$ depends on the type of crack and on its persistency, i.e., its degree of cracking $a/B$ or $b/W$. Figure 2 presents two basic configurations for partially jointed media. The choice of the most relevant geometry depends on the type and the degree of jointing. A summary of $F$ values is also presented in Fig. 2. For practice, values of 0.5 or higher are considered to correspond to completely broken-up media, i.e., the DI method becomes more applicable than the CFM method. For values of 0.1 or less, a tensile strength approach is more plausible than a Fracture Mechanics approach. $K_{Ic}$ is assumed depending on the mineralogy of the medium and the tensile strength $T$ or the unconfined compressive strength UCS. Furthermore, corrections are made to account for the effects of the loading rate and the in-situ stress field. The in-situ fracture toughness $K_{Ic, ins}$ is based on a linear regression of available literature data as follows:

$$K_{Ic, ins} = (0.008 \text{ to } 0.010) \cdot \text{UCS} + (0.054 \cdot \sigma_c) + 0.42$$

in which $\sigma_c$ represents the confinement horizontal in-situ stress and $T$, UCS and $\sigma_c$ are in MPa. Brittle joint propagation happens for $K_{Ic} > K_{Ic, ins}$. If this is not the case, joint propagation needs a certain time to happen. This is expressed by:

$$\frac{dL}{dN} = C_r \cdot (\Delta K_{Ic} / K_{Ic})^{m_r}$$

in which $N$ is the number of pressure cycles. $C_r$ and $m_r$ are material parameters that are determined by fatigue tests and $\Delta K_{Ic}$ is the difference of maximum and minimum stress intensity factors at the joint tip. To implement time-dependent joint propagation into a comprehensive engineering model, $m_r$ and $C_r$ have to be known. They represent the vulnerability of the medium to fatigue and may be derived from available literature data. These values express qualitative differences in sensitivity and no absolute values. Hence, any application should be based on appropriate calibration. A first-hand calibration for granite (Cahora-Bassa Dam; Bollaert, 2002) resulted in $C_r = 1E^-7$ for $m_r = 10$.

The fourth dynamic loading parameter is the maximum dynamic impulsion $C_{Ic}^{max}$ in an open-end rock joint (underneath a single block), obtained by Newton’s 2nd law:

$$I = \int \left( F_u - F_o - G_b - F_sh \right) \cdot dt = m \cdot V_{\Delta pulse}$$

in which $F_u$ and $F_o$ are the forces under and over the block, $G_b$ is the immerged weight of the block and $F_{sh}$ represents the shear and interlocking forces. The maximum net impulsion $I_{max}$ is defined as the product of a net force and a time period. The force is firstly transformed into a pressure. This pressure is then divided by the incoming kinetic energy $V^2/2g$. This results in a net uplift pressure coefficient $C_{up}$. The time period is non-dimensionalized by the travel period characteristic for pressure waves inside joints, i.e., $T = 2 \cdot Li/c$. This results in a time coefficient $T_{up}$. Hence, $C_I$ is defined by the product $C_{up} \cdot T_{up} = V^2 \cdot L/g \cdot c$ [m·s]. The maximum net impulsion $I_{max}$ is obtained by multiplication of $C_I$ by $V^2 \cdot L/g \cdot c$. The $C_{up}$ value was measured close to 0.35.

Failure of a block is expressed by the displacement it undergoes due to the net impulsion $C_I$. This is obtained by transformation of velocity into uplift displacement $h_{up}$. The net uplift displacement necessary to eject a block is difficult to define. The necessary
displacement is a model parameter that needs to be calibrated. A first-hand calibration on Cahora-Bassa Dam (Bollaert, 2002) resulted in a critical net uplift displacement of 0.20.

3 Case study: Kariba Dam scour hole (Zimbabwe)

3.1 Introduction

The CSM model has been applied to the Kariba Dam scour hole. Since 1962, spillway discharges from Kariba Dam have eroded a scour hole into the gneiss rock, which extends about 80 m below the initial river bed (Mason & Arumugam, 1985). A detailed analysis of the annual discharges and related scour formation allowed calibrating the CSM model and predicting future scour formation as a function of time. Especially the time-related parameters of the CSM model have been adapted to the long-duration observed prototype scour. Comparison has been made with calibration based on Cahora-Bassa Dam scour.

![Figure 3. Kariba Dam scour hole development as a function of time.](image)

3.2 Parameters

Kariba Dam is a double curvature mass concrete arch dam 130 m high situated on the Zambesi river between Zambia and Zimbabwe. After dam construction in 1959, a large scour hole quickly formed in the downstream fractured rock. Details are available on the reservoir levels, spillway discharges and tailwater levels, as well as on the average time duration of floods. Furthermore, after each major flood period between 1962 and 1981, a detailed bathymetric survey of the scour hole has been carried out.

The spillway consists of 6 rectangular gate openings of 8.8 m by 9.1 m, for a total discharge of about 9'500 m$^3$/s. The gate lips are situated at around 456.5 m a.s.l. The minimum and maximum reservoir operating levels are 475.5 and 487.5 m a.s.l. The downstream tailwater level is situated between 390 and 410 m a.s.l., depending on the number of gates functioning. An average value of 400 m a.s.l. has been assumed for the computations. The net head difference results in typical jet outlet velocities of 21.5 m/s. Scour formation in the rock mass reached a level of 306 m a.s.l. in 1981, i.e. about 80 m down the initial bedrock level. The rock mass is sound gneiss with a degree of fracturing.
that is not known precisely. Without further noticeable information on rock mass quality, the computations have been performed for a set of conservative, average and beneficial parametric assumptions. The spillway discharges are generally performed for varying gate openings and operations, as a function of already formed scour. This results in complex and varying hydraulics. In the following, a 2D simplified approach is considered, assuming only one jet and a (reasonable) average gate opening of 75%. The time durations of the floods also vary from year to year. Nevertheless, it is well known that the flood season generally takes several months in this region. Hence, an average duration of 3 months or 90 days per year is assumed for the scour computations.

<table>
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<tr>
<th>Property</th>
<th>Symbol</th>
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<th>AVERAGE</th>
<th>BENEF</th>
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<td>m</td>
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<tr>
<td>Total number of joint sets</td>
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<td>100</td>
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Table 1: Rock mass properties under different parametric assumptions

3.3 Calibration of fatigue parameters

Based on the parametric assumptions above, computed scour formation has been calibrated in order to match the in-situ scour. The calibration parameters are the fatigue coefficients \( C_r \) and \( m_r \). These define the time-dependency of the scour formation and express the resistance of the medium against joint propagation by fatigue. Figure 4 presents results of computed versus in-situ estimated flood durations between 1962 and 1981, as well as corresponding appropriate combinations of \( C_r \)-\( m_r \) values for average parametric assumptions. In-situ scour formation is based on Mason & Arumugam (1985).

Figure 4. a) Comparison of computed and in-situ estimated flood duration; b) \( C_r \)-\( m_r \) relationships as calibrated.
Good agreement is obtained between computed and in-situ estimated flood durations. The corresponding combinations of $C_r$-values are presented in Fig. 4b for all parametric assumptions. It has to be outlined that the computed results are of qualitative character.

4 Case study: Mississippi shale-rock (USA)

4.1 Introduction

In 1991, undisturbed shale samples were extracted from a bridge construction site on the Mississippi River (Nakato, 2002) and were tested at IIHR (Iowa) against erosion under prototype jet velocity and time conditions. The tests indicated that some scour potential exists at high velocities and procured the time evolution of the observed scour. The corresponding hydrodynamic and geomechanic characteristics have been introduced into the CSM model, which was able to reproduce the tested scour as a function of time.

4.2 Test facility

A test facility was constructed with a 5.1 cm diameter circular jet impinging onto a rock sample of about 0.60 m x 0.60 m x 0.40 m, under an angle with the horizontal of 55° (Fig. 5). Jet velocities of up to 4.57 m/s have been tested and different erosion patterns have been observed. The largest observed scour depth for the highest velocity was 15.2 cm after running for 13.5 hrs. Despite the qualitative character of erosion when compared to real-life situations at bridge piers (with large-scale eddies, 3D currents, etc.), the tests proved first of all that shale-rock is sensitive to time-dependent erosion formation due to flow velocities typical for turbulent flow around bridge piers. Second, the available time evolution of the observed scour formation in the shale rock makes it very interesting to calibrate the CSM model. This is especially relevant because of the weak intact strength of the shale rock when compared with typical rocks as encountered in dam engineering, which procures an additional calibration interest.

4.3 CSM calibration

The CSM model has been applied for laboratory tests with velocities of 1.85, 3.05 and 4.57 m/s. For both latter tests, significant scour has been observed during the tests. However, for the former test, no scour was observed even after 19 hours of discharge. The
CSM model calibrated $C_r$ and $m_r$ (fatigue) parameters in order to obtain scour formation as a function of time that is similar to the one observed in the laboratory. This scour formation is presented in Figure 6a for an estimated UCS strength of the shale-rock of 8 MPa and average parametric assumptions following Table 1.

![Graph showing comparison of computed and laboratory measured scour](image)

Figure 6. a) Comparison of computed and laboratory measured scour; b) $C_r$-$m_r$ relationships as calibrated.

It can be noticed that the model is able to simulate the laboratory observed scour very well for velocities of 3.05 and 4.57 m/s. Also, for the low velocity of 1.85 m/s, no scour was predicted by the model, which is in agreement with the laboratory observations. Second, Fig. 6b shows the calibrated combinations of $C_r$-$m_r$ values. These values are much lower than the ones previously obtained at Kariba Dam, which is in good agreement with typical values available in literature data for weak rocks (Bollaert, 2002 & 2004).

5 Conclusions

A new engineering model, the Comprehensive Scour Model (CSM), has been developed for evaluation of scour in fractured media. The model not only predicts the ultimate depth of scour but also the time evolution of scour formation. It makes use of Linear Elastic Fracture Mechanics to incorporate a fatigue law for the fractures and can be applied to any type of fractured medium, such as rock, concrete, strong clays, etc.

The present paper applies the new model to two completely different situations of fractured media that are subjected to turbulent high-velocity flow. The first situation considers the plunge pool scour formation behind Kariba Dam (Zimbabwe), while the second situation deals with scour formation in shale-rock on the Mississippi River, tested under prototype laboratory conditions. Both examples allowed calibrating the fatigue parameters of the model, which showed very good agreement with observed scour formations. Hence, once appropriately calibrated, the model is not only able to simulate past scour but also to predict future scour as a function of time duration of floods.

References


